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A MODEL FOR KINKING IN FIBER COMPOSITES—II. KINK BAND FORMATION

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Abstract—In Part II of this series, the critical strain at which a fully formed kink band can exist in a compressed fiber composite is calculated. This calculation is based on a comparison of the energies of the kinked and unkinked configurations, assuming both satisfy all relevant field equations. The critical strain is found to be on the order of $2\tau_s/E_L$, where τ_s is the longitudinal shear strength and E_L is the composite elastic modulus parallel to the fibers. In Part I, the strain to cause the fibers to break was found to be significantly higher, in general, closer to observed failure strains. This suggests that fiber breakage is the critical step in the process of kinking, not the actual formation of the band.

INTRODUCTION

Advanced fiber composites subjected to compressive loadings parallel to the fibers often fail with the development of kink bands. The present paper is the second of a two-part series dealing with two critical aspects of kinking in fiber composites. An idealized view of the failure sequence assumed here to be operative is shown schematically in Fig. 1. Fibers bow under the compressive load until they break; then, the kink band can form. Part I of this sequence was devoted to developing a means of computing the strain at which the fiber breaks can occur at the kink band boundary. Here, in Part II, we put forth a means of calculating the strain at which a fully-developed kink band, as shown in Fig. 1c, can exist in a compressed composite, *assuming the necessary fiber breaks are already present.* Within such a framework, the higher of these two critical strains is the strain at which the kink band will actually form.

As indicated in Part I, most theoretical modeling of compressive failure in fiber-reinforced composites has concentrated on micro-buckling. These models, in general, fail to account for three significant features of kinking: (i) the fibers break at the kink band boundaries; (ii) the width of kink bands is small compared with the specimen length; and (iii) there is a specific orientation relation between the kinked fibers and the kink band boundary. In Part I, the fiber breaks were associated with the substantial bending that

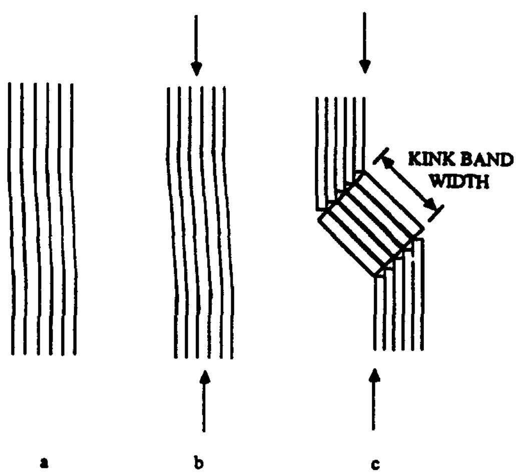

Fig. 1. Schematic of kinking process.

occurs during fiber micro-buckling. Some suggestions were also made as to the determinants of the kink width. Here, we will focus on the actual formation of the kink band; the model put forth sheds light on the basis for the observed orientation relation. Some attempts to model the formation of the actual band were made by Evans and Adler (1978), who also made a number of observations of kinking under a variety of loading conditions. On the basis of work and energy arguments, they give estimates for the angles of the kinked fibers and of the kink band boundary. However, they do not offer a specific means of calculating the overall stress or strain at which kinking can occur. Budiansky (1983) has employed a couple-stress theory (by smearing out the fibers) to arrive at estimates of the characteristic angles. Hahn (1987) has attempted to link micro-buckling with the ultimate kink failure.

The problem of the formation of a fully-developed kink band would appear to be vaguely similar to that of shear band development. Initially put forth by Hill (1962), and elaborated by Rice (1976), the dominant theoretical approach to shear banding has been to treat the localized plastic deformation as a solution bifurcation. In essence, one considers a solid subjected to a homogeneous deformation and, then, asks the following question: Can there be an incremental shearing of arbitrary magnitude within a band of material, while the material outside the band continues to deform in its original homogeneous fashion? The answer is yes, provided the equations governing incremental deformation are no longer elliptic. For many material laws, this will occur if the strain is sufficiently high (generally in the finite strain regime).

Such an approach is doomed to failure here, however, in that the material outside the kink bands is still linear elastic, judging by the strains at which kinking occurs. In fact, a very different approach is pursued here to model kink band formation. Instead of posing the problem incrementally and looking for a shearing of arbitrary magnitude, we look for a *single* kinked configuration that is *finitely* different from the unkinked state; details of the kink band as it evolves are not of concern. This seems appropriate as the details of the intermediate configurations are generally not known; most observations are of fully developed kink bands.

The particular procedure adopted herein is to assert that a fully developed kink band, described by appropriate geometric parameters, can exist at a given overall compressive stress provided two conditions are satisfied: (i) the kink band is in mechanical equilibrium with, and kinematically consistent with, the unkinked material; (ii) the energy associated with the kinked configuration is no higher than the energy of the unkinked configuration. The latter condition is necessary because we are *not* taking an incremental approach to kink band development. In the classical incremental approach to bifurcation problems, one can show the following: when the load has reached a level at which there exists a second incremental solution to the field equations, transition from the first solution to the second solution involves no change in energy. This cannot be asserted when the two solutions (kinked and unkinked) are finitely different; thus the need to enforce a non-positive change in energy.

ANALYSIS

Consider a unidirectional composite, which is subjected to a monotonically increasing compressive stress p ($p > 0$) acting parallel to the fibers. At each level of the stress p, one inquires as to whether the conditions for the formation of a kink band are satisfied. Here, the formation of a kink band means the following. Most of the composite remains elastic under a uniaxial compression p, except for a band of material ABCD (see Fig. 2a), which is characterized by its boundary orientation β. Within ABCD, the stress and strain are homogeneous, but different from outside the band (see Fig. 2b); nevertheless, continuity of traction and displacement across the band is maintained. The material in the kink band responds in a partially plastic fashion; the details of the constitutive description are taken up below. In addition, the energy change associated with going from the unkinked state to the kinked state must be non-positive.

One can view the final kinked configuration as being arrived at by the development, first, of a uniform field of infinitesimal elastic strains that are associated with the stress p,

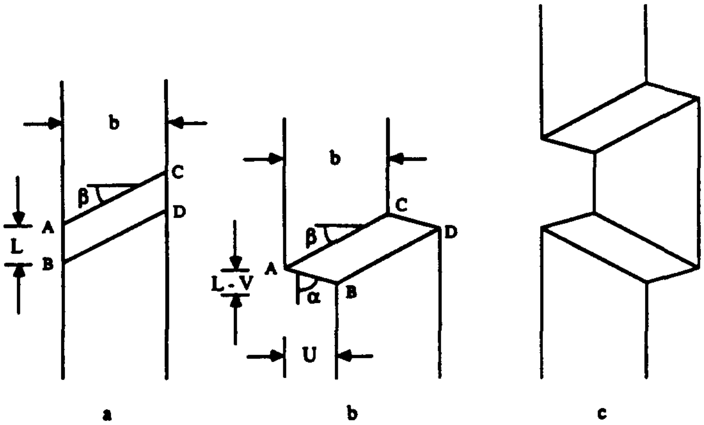

Fig. 2. Schematic of kink band formation and definition of kinematic parameters.

followed by the "kink deformation", which is a finite deformation in the kink band with the material outside the band remaining rigid. With this succession of events, the finite deformation in the kink band must involve no extension of the lines AC and BD. The homogeneous deformation of the kink band can, therefore, be described completely by the horizontal and vertical displacement U and V undergone by the point B. Note that the specimen can remain aligned if a second conjugate kink were to occur as shown in Fig. 2c. Only one kink need be considered, however, since the conditions for the formation of the kinks are the same. The kink band angle α (giving the final orientation of the fibers) is related to U and V by

$$\tan\alpha = \frac{U}{L-V}. \tag{1}$$

To impose traction continuity, as well as the energy criterion, it is necessary to invoke a constitutive law for the finitely deformed composite material making up the kink band. A completely general law for all multi-axial stress states is not proposed, however; nor is it necessary, as can be appreciated by viewing the kinematics of kink deformation as follows (see Fig. 3):

(i) A rigid rotation through an angle α.
(ii) Extensions parallel to and normal to the fiber direction.

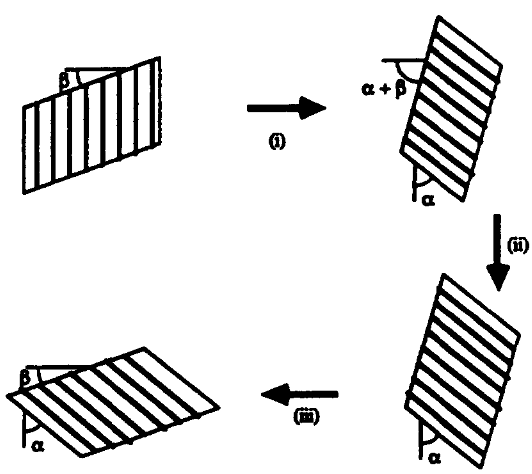

Fig. 3. Decomposition of kink band deformation.

(iii) A simple shearing which leaves the fibers in the same direction.

We now assume that the normal strains which develop parallel and normal to the fibers are small. A consideration of the results of our computations will show this assumption to be valid. Furthermore, it is assumed that the orthotropy (actually the material is transversely isotropic) which clearly prevails when the composite is deformed elastically continues to prevail for this finitely deformed state of infinitesimal normal strains and a finite longitudinal shear strain. Hence, the normal stresses due to the straining parallel and perpendicular to the fibers are unaffected by the shearing; they are related to the normal strains by the orthotropic linear elastic moduli of the unidirectional composite.

Assuming plane stress, one finds the relation to be

$$\varepsilon_f = \frac{1}{E_L}\sigma_f - \frac{\nu_T}{E_T}\sigma_t \tag{2a}$$

$$\varepsilon_t = -\frac{\nu_T}{E_T}\sigma_f + \frac{1}{E_T}\sigma_t \tag{2b}$$

where E_L, E_T and ν_T are the relevant moduli, σ_f and ε_f are the stress and strain parallel with the *final* fiber direction and σ_t and ε_t are in the direction transverse to the final fiber direction.

Consistent with the assumed orthotropy, the finite shearing must be independent of the normal strains; it is assumed to occur at a constant shear stress which is equal to the longitudinal shear strength of the composite τ_s. Since the constitutive law decouples the normal stresses from the shear strains and the shear stresses from the normal strains, the stresses in the final kinked state and the work done to reach the final kinked state are essentially independent of the path taken to reach the final state. In particular, it is not necessary for *all* the kink deformation to occur *after* the uniform deformation associated with p, as assumed above in deducing the kinematics.

The total extensional strains in the kink band are given by

$$\varepsilon_f = \frac{-p}{E_L} + \frac{1}{2}\left[\frac{U^2}{L^2} - 2\frac{V}{L} + \frac{V^2}{L^2}\right] \tag{3a}$$

$$\varepsilon_t = \frac{\nu_T}{E_T}p + \frac{1}{2}\left[\left(\frac{\cos(\alpha-\beta)}{\cos\beta}\right)^2 - 1\right] \tag{3b}$$

where the first terms involving p are the pre-kinking strains, and the remaining terms are associated with the kink deformation. Note that the Lagrangian strain has been used for the kink deformation. This particular choice is not necessary, however, because the strains are assumed to be (and turn out to be) infinitesimal, which justifies the linear relation (2). It is only necessary to employ a strain measure which is unaffected by the finite rotation associated with U and V.

Through a transformation of stress components, one finds that continuity of tractions across the kink band boundary requires the stresses σ_f and σ_t to be related to p according to

$$-p\cos^2\beta = \sigma_f\cos^2(\alpha-\beta) + \sigma_t\sin^2(\alpha-\beta) - \tau_s\sin 2(\alpha-\beta) \tag{4a}$$

$$-\tfrac{1}{2}p\sin 2\beta = \tfrac{1}{2}(\sigma_t-\sigma_f)\sin 2(\alpha-\beta) - \tau_s\cos 2(\alpha-\beta). \tag{4b}$$

The energy difference per unit thickness between the kinked and unkinked configurations, ΔE_{tot}, may be expressed as the sum of three terms

$$\Delta E_{tot} = -pVb + \Delta E_{el} + \Delta W_p \tag{5}$$

where $-pVb$ is the work done by the external load (b is the in-plane width of the specimen),

ΔE_{el} is the change in elastic energy in the band, and ΔW_p is the plastic work done in the kink band (by shearing). Ignoring any elastic portion of the shear deformation, ΔW_p is found to be

$$\Delta W_p = \tau_s bL \tan\gamma \tag{6}$$

where the shear strain angle, γ, is related to α and β according to

$$\tan\gamma = \tan\beta + \tan(\alpha - \beta). \tag{7}$$

One can finally write the energy change in the form

$$\frac{\Delta E_{tot}}{E_L bL} = -\varepsilon_0 \frac{V}{L} + \frac{1}{2}\frac{(\sigma_f \varepsilon_f + \sigma_t \varepsilon_t)}{E_L} - \frac{1}{2}\varepsilon_0^2 + \frac{\tau_s}{E_L}\tan\gamma \tag{8}$$

where ε_0 is defined by

$$\varepsilon_0 = \frac{p}{E_L}. \tag{9}$$

Given the material parameters ν_T, E_T/E_L and τ_s, the following procedure was followed to determine when conditions for the emergence of a kink band are satisfied. First, α and β were fixed, and p (or rather ε_0) was eliminated between equations (4) yielding a quadratic equation for V. (This required the elimination of σ_f and σ_t using (3) and U using (1).) Provided a real root existed, then ε_0 was determined; if there were not a real root, then no kink band would be possible for those values of α and β. If the ε_0 so determined were real, but negative (signaling an applied tension), then no kink band would be possible. Provided ε_0 was positive, the change in energy was evaluated. Kink bands are predicted to occur when the change in energy was non-positive.

RESULTS AND DISCUSSION

The predictions of the present theory of kink band formation are now considered using material properties that are typical of carbon- and glass-reinforced plastics. In particular, we took $\nu_T = 0.019$, $E_L = 138$ GPa, $E_T = 8.96$ GPa and $\tau_s = 93$ MPa, which are typical of a carbon-reinforced epoxy (see Tsai, 1987). Figure 4 shows the stress p that is in mechanical equilibrium with the kink band described by parameters α and β. Clearly, insofar as equilibrium and kinematics alone are concerned, a kink band can form at virtually any angle given the right load. However, not all these kink bands correspond to a decrease in energy in going from the unkinked configuration to the kinked configuration. It was found

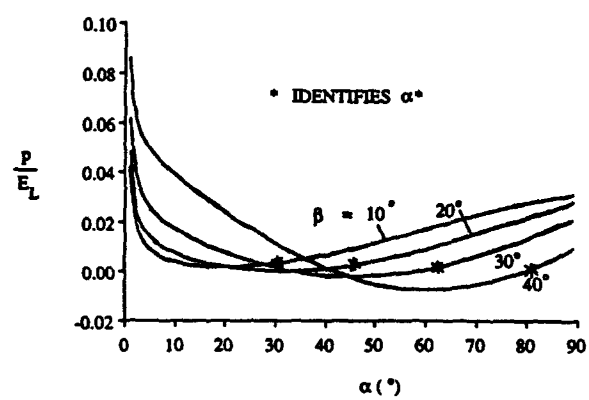

Fig. 4. Normalized pressure to form a kink band for various angles α and β, with critical value α^* indicated ($\nu_T = 0.019$, $E_L = 138$ GPa, $E_T = 8.96$ GPa, $\tau_s = 93$ MPa).

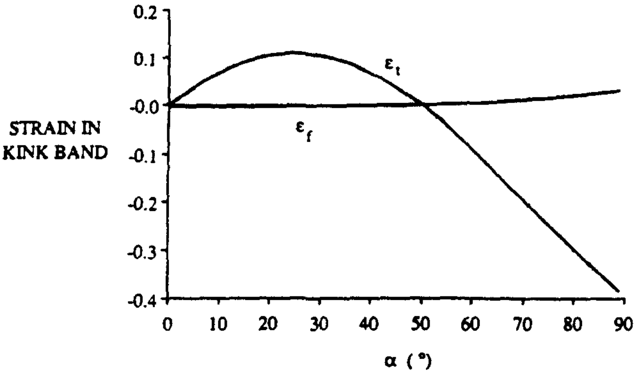

Fig. 5. Kink band strains as a function of kink angle α ($\beta = 25^\circ$; $\nu_T = 0.019$, $E_L = 138$ GPa, $E_T = 8.96$ GPa, $\tau_s = 93$ MPa).

that the energy change is non-positive (favoring kinking) only for values of α which exceeded some critical value $\alpha^*(\beta)$. The values of α^* are indicated by the asterisks on the curves in Fig. 4. The critical value $\alpha^*(\beta)$ is always greater than 2β, but approaches 2β as $\beta \to 45°$. (In fact, once σ_f and σ_t in (4) have been eliminated using (2) and (3), it may be seen that equilibrium cannot be satisfied if α is *precisely* equal to 2β, unless $\beta = 45°$.) Note that once α exceeds the critical value α^*, the load for kinking increases monotonically with α. Therefore, for fixed β, this provides a critical kinking stress, which corresponds to a kink described by $\alpha = \alpha^*$. It should be noted that this is consistent with observations by Evans and Adler (1978) and Weaver and Williams (1975) that α is slightly in excess of 2β.

That the critical angle for kinking α^* is approximately equal to 2β is rather fortunate for the present theory; combinations of α and β that deviate greatly from $\alpha \approx 2\beta$ correspond to large values of the normal strain ε_t, as may be seen from Fig. 5. In the vicinity of $\alpha = 2\beta$, ε_t is small, and it increases as α deviates from 2β. The source of the large values of ε_t is obvious from (3b); they violate our assumption of a linear elastic response to the normal strains parallel and perpendicular to the fiber. In any event, because $\alpha^* \approx 2\beta$, the kinking criterion of $\alpha = \alpha^*$, with its attendant p, satisfies all the assumptions that underlie the theory. The critical value of p is plotted in Fig. 6 as a function of β; also plotted are the results of computations carried out for material properties appropriate to a glass-reinforced epoxy ($\nu_T = 0.056$, $E_L = 38.6$ GPa, $E_T = 8.27$ GPa and $\tau_s = 72$ MPa). The critical value of p can be seen to approach $2\tau_s$ as $\beta \to 45°$. This was found, in fact, to be the case for a wide range of material parameters.

The present theory does predict the observed relation between α and β; however, it also predicts that kink bands will appear at successively lower pressures as β increases. Though there is considerable scatter in the data, observed values of β are roughly in the range $15° < \beta < 30°$. Perhaps the problem with the present theory is its assumption of a

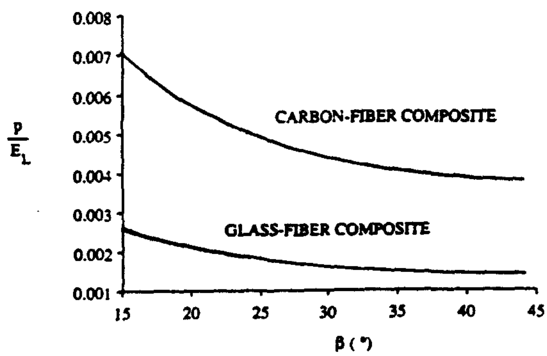

Fig. 6. Minimum pressure to form a kink band for two composites (carbon-fiber: $\nu_T = 0.019$, $E_L = 138$ GPa, $E_T = 8.96$ GPa, $\tau_s = 93$ MPa; glass-fiber: $\nu_T = 0.056$, $E_L = 38.6$ GPa, $E_T = 8.27$ GPa, $\tau_s = 72$ MPa).

neglible transition zone from kinked to unkinked material. In fact, one can expect that there is a transition zone which is on the order of the fiber diameter. Furthermore, the matrix material in this transition zone would be subjected to extremely intense deformation if β were to be large (approaching 45°). Even though the transition zone has been neglected, the equilibrium relation as currently formulated (connecting the unkinked material and the *homogeneously* kinked material) is still acceptable, as can be seen by noting that the tractions must be continuous across the transition zone as well. However, the energy of the transition zone is being neglected, and its contribution may be non-negligible in that the kink band width is only one order of magnitude greater than the fiber diameter. With proper accounting for the transition zone, the curves of critical pressure in Fig. 6 might increase again as β exceeded some value.

CONCLUSIONS

In this second part of a two-part sequence the formation of a fully developed kink band has been studied. The observed relation between the kink band boundary (β) and the fibers in the kink band (α) has been predicted, though the correct range of observed β is not predicted. We have found that the overall strain at which a kink band can exist—*presuming the existence of the requisite fiber breaks at the kink band boundary*—is slightly in excess of $2\tau_s/E_L$. In general, this strain is significantly less than the fiber breaking strain found in Part I, which was close to the observed compressive failure strains. This appears to support the conclusion arrived at in Part I, that the breaking of the fibers is the limiting step in the formation of the kink band. Hence, the present modeling effort suggests the following scenario for kink band formation. Initially, the compressive load causes the fibers in a misaligned bundle to bow. With sufficient load, the bowing leads to fiber fracture. Once fiber fracture occurs, then conditions for kink band formation are already met (and exceeded). Hence, the kink band can form immediately upon fiber breakage.

Unfortunately, this is not the whole story. For example, partially formed kinks have been observed by Evans and Adler (1978); Chaplin (1977) has observed propagating kink bands. The present model does not account for the fact that the kinking of the fibers in the misaligned bundle must somehow communicate their effect to the remainder of the composite. In addition, compressive failure by kinking often initiates at the free surface or at cutouts. Initiation at such imperfections might lead to qualitatively different conclusions. For example, as a kink band propagates, it may aid in the breaking of fibers. Carefully controlled experiments that could give insight into these processes would be of great assistance in future modeling efforts.

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